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Mathematical models to optimize the use of forest land

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Abstract Large share of that land covered with forest vegetation occupies, both at planetary and national level, determine a very high importance to its administration. Forests as a whole are the main means of environmental protection and cover a significant land area. Forest management planning is a science of organization, modeling, and forest management depending on the objectives set by policy makers at a time. This way of organizing and trying to ensure a continuous harvest of wood faced great difficulties for private forests due to relatively small areas and the high degree of territorial dispersion and of properties and the divergent interests of holders. In order to optimize the exploitation of forest instruction were issued as prescribed by the forest management plans, as follows: general plan of operation, especially operating plan, the plan of improvement works, works of afforestation plan, and the plan of construction. In present, a special chapter on forest protection, including measures necessary to ensure stability or ecological restoration of this fund, where applicable, was added. Note that, initially, in the process of recovery and optimize forest resources, forest management plans were focused especially on the upper side turning wood as the main resource of the forest and then, so far, the data collected from field were becoming more complex in order to identify and higher capitalization of all forest resources.

Key words

graph, forest management plan, Hamiltonian path

Hamiltonian paths and cycles

Forest management plans can raise several problems whose solution needs their modeling in as graphs and determination in these graphs of some Hamiltonian paths or circuits (cycles) of minimum length (Bândiu, 1999). In a graph G, a way is Hamiltonian if it passes through all nodes of the graph at least once (Skiena 1990, p. 196). If the graph contains n nodes, a Hamiltonian way length is n-1(equal to the number of arcs of the road). A necessary condition for a graph to be a Hamiltonian way is that the graph is completely (between any two of its nodes must be at least one arc, regardless of its meaning). A circuit is Hamiltonian if it passes at least once through all nodes of the graph, except of one node, which is both the origin and final point of the cycle (Iwamoto and Toussaint 1994). A graph possessing a Hamiltonian cycle is said to be a Hamiltonian graph. The Hamiltonian cycle is named after Sir William Rowan Hamilton, who devised a puzzle in which such a path along the polyhedron edges of an dodecahedron was sought (the Icosian game) and can be used to optimize different surfaces (Chalaturnyk, 2008).

Based on the following map of forest management plan we intend to create a mathematical model using the graph theory to optimize this forestland administration.

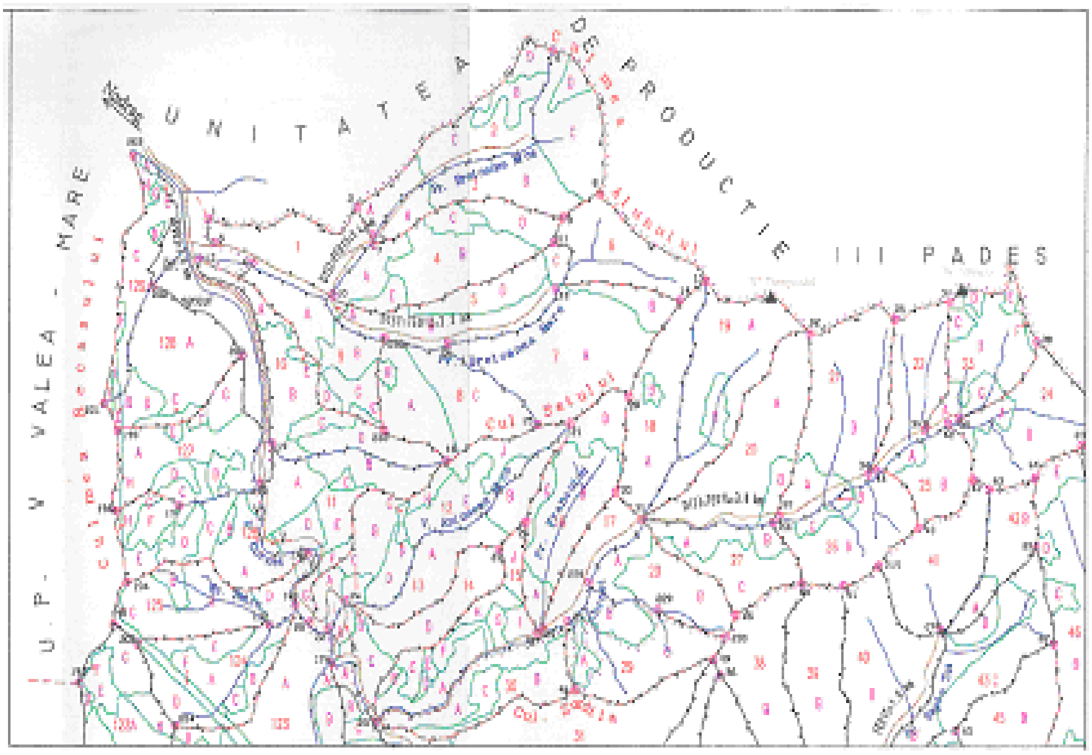

Fig 1 Map of a forestland

Distance between two peaks

Given two peaks X_i and X_j of one graph $G=(X,\Gamma)$. On call "closure" from X_i to X_j the number of vertices of minimum path length from X_i to X_j. Closure is noted with d (X_i, X_j). When there is no path from X_i to X_j on take $(X_i, X_j)=\infty$.

In order to find the path or paths that correspond to the closure between two peaks the following, very simple algorithm can be applied: on assign each peak X_i systematically, a rate ***m*** equal to the length of the shortest path that leads from X_0 (origin) to X_i and so far reaching in X_n (destination).

Minimum value road (weighted length)

Given a graph $G=(X,\bigcup)$, each vertex $u \in \bigcup\ i$ is associated with on number $l(u)\geq 0$ named „value" of **u** ; on require to find a way μ starting from the peak $A \in X$ to thepeak $B \in X$ so that the total $l(u)=\sum_{u\in\mu} l(u)$ is minimal.

There are several algorithms to solve this problem. We propose the Ford Algorithm to be used in this case:

1) Each peak X_i has an index λ_i. On start with $\lambda_0 = 0$ and $\lambda_i = +\infty$ if $i \neq 0$.
2) On look for one arc (X_i, X_j) so that : $\lambda_j - \lambda_i > l(X_i, X_j)$. Then λ_j is replaced by $\lambda_j > 0$ if $j \neq 0$. Continue until no arc will allow indices decrease λ_i.
3) There is a peak X_{p_1} so: $\lambda_n - \lambda_{p_1} = l(X_{p_1}, X_n)$. Thus, the map of forest management plan can be optimized by the model below.

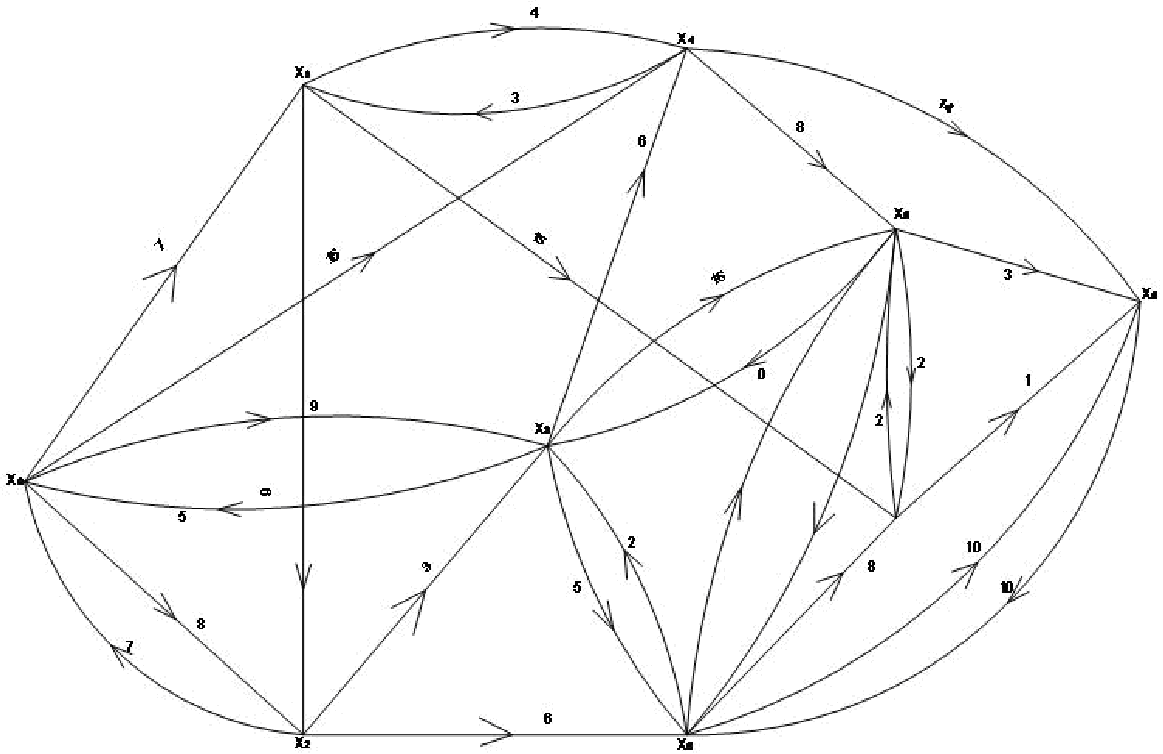

Fig. 2 Hamiltonian graph for optimization of forest management plan

Due to λ_n gradually decreasing during the procedure and X_{p_1} is the last peak used to decrease λ_n on take X_{p_1} so that: $\lambda_n - \lambda_{p_2} = l(X_{p_2} - X_{p_1})...etc;$ and the string $\lambda_n, \lambda_{p_1}, \lambda_{p_2},...$ is strictly decreasing , we will have at some point: $X_{p_{k+1}} = X_0$.

In this case λ_n is the value of the minimal path from X_0 to X_n, and this path is: $\mu = \lfloor X_0, X_{p_{k+1}}, X_{p_K},..., X_{p_1}, X_n \rfloor$.

Bellman – Kalaba Algorithm

The method of quasilinearization developed by Bellman and Kalaba provides an explicit approach for obtaining approximate solutions to nonlinear differential equations and it gives pointwise convergence lower estimates of the solution of the given problem provided the function involved is convex (Bellman and Kalaba, 1965). This is a variant of the previous algorithm presented using dynamic programming. It is based on the following property: “Every path that has at least **r** arcs and is minimal consists of partial paths containing at most **k** arcs (k ≤ r) that are also minimal.”

Another notation can be used. Assume that $c_{ij} \geq 0$ is a value associated to the vertex (X_i, X_j) for each $(X_i, X_j) \notin \bigcup$ is taken $c_{ij} = \infty$ and for every(X_i, X_j) on take $c_{ij} = 0$. We want to find a path: $\mu = \lfloor X_0, X_{i_1}, X_{i_2},..., X_{i_k}, X_n \rfloor$, such that: $c_{0i_1} + c_{i_1i_2} + c_{i_2i_3} + ... + c_{i_kn}$ is minimal.

Maximum value road

Bellman-Kalaba algorithm can be applied also when a maximum value road is searched, but with an obvious restriction: graph should not contain any circuits or loops, otherwise, as the length of the road (number of adjacent edges) may not be finite, value of this road is not bounded.

Conclusion

Mathematical modeling using graphs theory allows achievement of optimal forest management plans depending on the requests and determines the minimal number of nodes or the minimal Hamiltonian path between two given points.

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